Reading Time: An initial **2 minutes** to view **BOTH** sections

MATHEMATICS METHODS : UNITS 3 & 4, 2023

Test 2 – Integrals, FTC and Exponential Functions (10%)

3.2.4, 3.2.6 – 3.2.22, 3.1.1 – 3.1.4, 3.1.9(exponentials only)

PM

Time Allowed 25 minutes	**First Name**	**Surname**	**Marks** 26 marks

Circle your Teacher's Name: Mrs Alvaro, Ms Chua, Mrs Fraser-Jones, Mrs Greenaway, Mr Luzuk, Mrs Murray, Ms Narendranathan, Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Not Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART A – CALCULATOR FREE

QUESTION 1 **(1, 1, 3: 5 marks)**

Differentiate the following, giving your answer in simplest form:

a) $y = 2e^{x^2+x}$

b) $\int_{-2}^{x} \frac{e^t}{t^3}\, dt$

c) $f(x) = \frac{e^{3x^4}}{2x^2}$

QUESTION 2 **(2, 2, 3 - 7 marks)**

Determine the following:

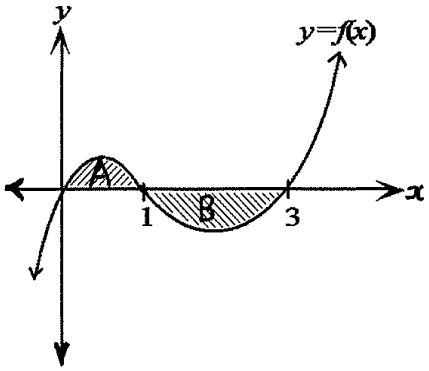
a) $\int 12x^3 e^{x^4+5}\, dx$

b) $\int_1^0 \frac{d}{dx}[4\pi x \cdot e^{3x}]dx$

c) $\int 3x(x^2-1)^4 dx$

QUESTION 3 **(1, 1, 2, 2 – 6 marks)**

The graph of $y = f(x)$ is shown below. The area of the shaded region *A* is 12 square units and of region *B* is 24 square units.



Evaluate the following.

a) $\int_0^1 2\, f(x)dx$

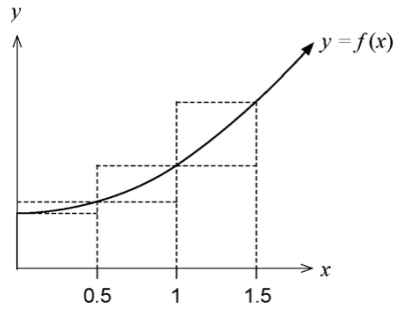
b) $\int_0^3 f(x)dx$

c) $\int_0^1 [\,1 + f(x)]\, dx$

d) $\int_1^3 f'(x)dx$

QUESTION 4 **(3 marks)**

Consider the function $f(x)$ shown graphed below. The table gives the value of the function at the given x values.

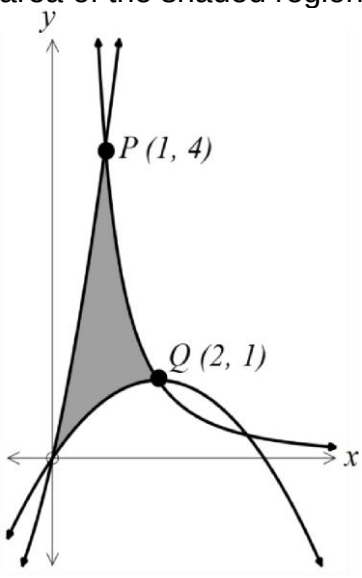


x	0	0.5	1	1.5
$f(x)$	18	19	22	27

By considering the areas of the rectangles shown, demonstrate and explain why $29.5 < \int_0^{1.5} f(x)dx < 34$.

QUESTION 5 **(5 marks)**

The origin, O, and the points P and Q are the vertices of the curved 'triangle' which is shaded in the diagram. The sides lie on curves with equations $y = x(x + 3)$, $y = x - \frac{x^2}{4}$ and $y = \frac{4}{x^2}$. Calculate the area of the shaded region.



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MATHEMATICS METHODS : UNITS 3 & 4, 2023

Test 2 – Integrals, FTC and Exponential Functions (10%)

3.2.4, 3.2.6 – 3.2.22, 3.1.1 – 3.1.4, 3.1.9(exponentials only)

PM

Time Allowed 20 minutes	**First Name**	**Surname**	**Marks** 19 marks

Circle your Teacher's Name: Mrs Alvaro, Ms Chua, Mrs Fraser-Jones, Mrs Greenaway, Mr Luzuk, Mrs Murray, Ms Narendranathan, Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART B – CALCULATOR ASSUMED

QUESTION 6 **(3, 2, 2: 7 marks)**

In January 1995 the purebred dingo population on Fraser Island was 300. The population, P, since then can be modelled by:

$$P = 80 + Ae^{kt}$$

where A and k are constants, and t is the number of years since January 1995.

a) Show that $\frac{dP}{dt} = k(P - 80)$

b) In January 2015 it was found that the purebred population had dropped to 162. Show that the purebred dingo population is decreasing at an annual rate of approximately 5% per year.

c) Assuming this pattern continues, what will the purebred dingo population be in January 2050?

QUESTION 7 **(2 marks)**

An oil storage tank ruptures at a time $t = 0$ and oil leaks from the tank at a rate of $r(t) = -100e^{-0.01t}$ litres per minute. How much oil leaks out during the third hour?

QUESTION 8 **(4 marks)**

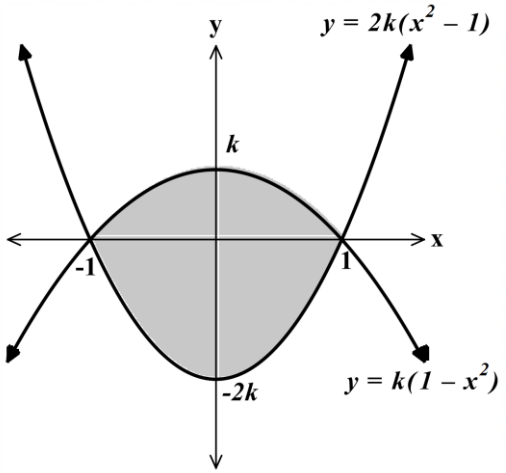
A product is sold such that the price per unit is given by $p = -3x^2 + 600x$ dollars when x units are sold. Find the marginal revenue at $x = 300$ units and interpret the result.

QUESTION 9 **(3 marks)**

A particle starts from rest at the origin and moves in a straight line with an acceleration of $a(t) = t^2 + e^{t-1} - 5\ m/s^2$. What is the total distance travelled in the 2nd second, to the nearest m?

QUESTION 10 **(3 marks)**

The shaded region shown is enclosed by two parabolas, each with x-intercepts at $x = -1$ and $x = 1$. Given that the area of the shaded region is $8\ units2$, find the value of k, where $k > 0$.



END OF TEST